

formula, with the differences by the two methods. - The author remarks that the first feature which presents itself, in running the eye over this table, is the enormous amount of the depression of the wet-bulb compared with our European experience. In March, the mean depression at no hour was less than  $14^{\circ}8$  at 7 A.M., increasing to  $29^{\circ}6$  at 3 P.M.; in April, the mean depression was never less than  $17^{\circ}3$  at 7 A.M., increasing to  $29^{\circ}9$  at 3 P.M.; and many observations necessarily much exceeded the maxima means. The next feature is the increase of the mean depression with that of the mean temperature, from 6 A.M. until 3 P.M., and then a decline with the decline of temperature until 9 P.M.; but not in the same ratio as the increase in the morning. With reference to the practical application of these observations with a view to determine the amount of moisture in the atmosphere, or to fix the dew-point, the author remarks that it will be seen from this table that Mr. Glaisher's factors give a higher dew-point than Dr. Apjohn's formula, varying in March from  $6^{\circ}1$  at 8 A.M. to  $11^{\circ}9$  at 6 P.M., and in April from  $5^{\circ}6$  at 7 A.M. to  $10^{\circ}4$  at 9 P.M.; and these varying discrepancies do not appear to have gradations of increment or decrement dependent upon increase or diminution of mean temperature, or increase or decrease of the depression of the wet-bulb. These remarks apply to the means of the observations; but with reference to isolated observations, the discrepancies by the two methods become much greater. On the 9th of April, at 8 P.M., the temperature of the air being  $97^{\circ}$ , the wet-bulb with a moderate draught  $60^{\circ}5$ , and with a strong draught  $60^{\circ}$ , the depressions were respectively  $36^{\circ}5$  and  $37^{\circ}$ , and the dew-point for the latter depression determined by Mr. Glaisher's factors would be  $41^{\circ}5$ , and  $12^{\circ}6$  by Dr. Apjohn's formula. In illustration of this part of the subject the author gives an extract of a letter from General Cullen, from which it appears that at Cochin on the Malabar coast, the temperature of the air being  $96^{\circ}$ , the wet-bulb  $61^{\circ}$ , the dew-point by Jones's hygrometer  $38^{\circ}$ , the dew-point by Mr. Glaisher's factors would be  $43^{\circ}5$ , and by Dr. Apjohn's formula  $22^{\circ}1$ .

13. "On a General Law of Density in saturated Vapours." By J. J. Waterston, Esq. Communicated by Lieut. Colonel Sabine, R.A., V.P. and Treas. R.S. &c. Received June 19, 1851.

The author of this paper commences by stating that the relation between the pressure and temperature of vapours in contact with their generating liquids has been expressed by a variety of empirical formulæ, which, although convenient for practical purposes, do not claim to represent any general law; and that some years ago, while examining a mathematical theory of gases, he endeavoured to find out, from the experiments of the French Academy, whether the density of steam in contact with water followed any distinct law with reference to the temperature measured from the zero of gaseous tension (situated at  $-461^{\circ}$  Fahr. by Rudberg's experiments, confirmed by Magnus and Regnault). To avoid circumlocution, he calls temperatures measured from this zero G temperatures, and observes

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that if  $t$  represents the G temperature,  $\Delta$  the density of a gas or vapour, and  $p$  its elastic force, the equation

$$t\Delta = p$$

represents the well-known laws of Marriotte and of Dalton and Gay-Lussac. He then states that, as the function which expresses a general relation between  $p$  and  $t$ , in vapours, must include a more simple function expressing a general relation between  $\Delta$  and  $t$ , the

proper course seemed to be to tabulate the quotients  $\frac{p}{t}$  from the experiments of the Academy, and to project them in a curve. For reasons connected with the *vis viva* theory of gases, which represents the G temperature as a square quantity, he projected these quotients or densities as ordinates, to the square root of the G temperatures as abscissæ; and found that the curve traced out was of the parabolic kind, but of a high order. Considering the density as a cubic quantity, the cube roots of the densities were set off as ordinates to the same abscissæ, and the author was gratified to find that the resulting curve was the Conic Parabola. To ascertain whether this was accurately the case, the square roots of these ordinates, corresponding to the sixth roots of the densities, were set off to the same abscissæ, that is the square roots of the G temperatures. The result is shown in a chart, in which, as the author observes, the points determined from the observations range with great precision in a straight line, any slight divergence being sometimes to the right and sometimes to the left; precisely as might be expected from small errors of observation. Other series of experiments on steam were projected in a similar manner, and it was found that, although no two exactly agreed with each other, each set ranged in a straight line nearly. The vapours of ether, alcohol and sulphuret of carbon, were found to conform to the same law, as were likewise M. Avogadro's observations on the vapour of mercury, and Faraday's experiments on liquified gases (Phil. Trans. 1845). Of these last olefiant gas is remarkably in accordance with the law, as are nitrous oxide, ammonia, cyanogen, sulphurous acid, and carbonic acid at the upper part of its range; but muriatic acid, sulphuretted and arseniuretted hydrogen, do not show the same regularity.

The co-ordinates of the points being the square root of the G temperatures and the sixth root of the densities, the equation to the straight line which passes through the points expresses the sixth root of the density in terms of the G temperature. The constants to be determined in this equation are the inclination of the straight line to the axis of  $x$  or that on which  $\sqrt{t}$  is measured, and the distance from the origin at which it cuts this axis, calling the cotangent of this angle  $h$ , and this distance  $g$ ,  $\Delta_1$ ,  $\Delta_2$  densities at G temperatures  $t_1$ ,  $t_2$ ,

$$h = \frac{\sqrt[6]{t_2} - \sqrt[6]{t_1}}{\sqrt[6]{\Delta_2} - \sqrt[6]{\Delta_1}} \text{ and } g = \sqrt[6]{t_1} - h \sqrt[6]{\Delta_1}.$$

The constants  $g$  and  $h$  being thus determined from two observations, the equation for the density at any other G temperature is

$$\Delta = \left\{ \frac{\sqrt{t-g}}{h} \right\}^6;$$

and for the pressure

$$p = \left\{ \frac{\sqrt{t-g}}{h} \right\}^6 t.$$

The several equations, with the numerical values of the constants  $g$  and  $h$ , for the series of observations previously referred to and represented on the chart, are then given, the  $G$  temperatures being in degrees of Fahrenheit's scale, and the values of  $h$  being calculated so as to give the pressure in inches of mercury.

The author remarks that the observations on the vapour of water below  $80^\circ$  show a small excess of density above what is required by the line corresponding to those at higher temperatures; and that it is a curious circumstance that the law of expansibility of water also becomes disturbed at about the same temperature. In proof of this, the observations of M. Despretz (*Ann. de Chim.* vol. lxx.) being projected, by making the volume the ordinate to the square root of the  $G$  temperature as abscissa, these observations above  $25^\circ$  C. or  $77^\circ$  F. give in the most exact manner a conic parabola; but below  $77^\circ$  they no longer give that curve.

The equation to the parabola for temperatures above  $77^\circ$  F. is  $\alpha(v-\theta) = (\sqrt{t}-\phi)^2$ , in which  $v$  is the volume at the  $G$  temperature  $t$ , in terms of its volume unity at  $39^\circ\cdot2$  F. or  $4^\circ$  C. (its point of maximum density),  $\alpha=352\cdot38$ ,  $\theta=.99872$ , and  $\phi=21\cdot977$  or  $\phi^2=483^\circ$ .

The law of the increase of density and temperature in saturated vapours having a certain analogy with the law of increase of density and temperature in air while suddenly compressed or dilated, the author next discusses the latter subject in a manner similar to that in which he had discussed the former. From this discussion he draws the following conclusions:—

1. When air is compressed or dilated, the  $G$  temperature varies as the cube root of the density; and the tension as the 4th power of the  $G$  temperature, or the cube root of the 4th power of the density.

2. The mechanical force exerted by a given quantity of air while expanding from one density to another, is proportional to the difference of the cube roots of these densities, or to the difference of their  $G$  temperatures: hence the fall of temperature is proportional to the force expended.

3. The mechanical force exerted upon a given quantity of air, while compressing it from one density to another, is proportional to the difference of the cube roots of these densities, or to the difference of their  $G$  temperatures: hence the rise of temperature is proportional to the force exerted.

4. The total mechanical force exerted by a volume of air of a given tension, while expanding indefinitely, is equal to that tension acting through three times the volume.

5. The total mechanical force exerted by a volume of air while expanding indefinitely is proportional to its  $G$  temperature.

6. A given quantity of air while expanding, under a constant pressure, from one temperature to another, exerts a mechanical force equivalent to one-third the difference of temperature; and the quantity of heat required to change the temperature of air under a constant pressure, is four-thirds of that required to effect the same change of temperature with a constant volume.

The author concludes by observing that it is singular that these simple and, he considers, important deductions from MM. Gay-Lussac and Welter's experiments, have been overlooked by the eminent mathematicians who have elaborately discussed this subject. The artificial position of the zero-point on the ordinary scales of temperature may perhaps account for this by its tendency to confine our ideas. Dalton's and Gay-Lussac's law of expansion seems imperatively to have required that, in all computations having reference to gases and vapours, the temperature should have been reckoned from the zero of gaseous tension; yet it has not been so; and it is impossible to avoid the conclusion, that if it had been otherwise, if no other temperature but what we have had so often to refer to as the  $G$  temperature had been indicated in their analyses, we should have profited more by their labours, and been further advanced in the science of heat and elastic fluids.

The Society then adjourned over the vacation to Thursday the 20th November, 1851.